

# A Dynamic-Epistemic Logic for Mobile Structured Agents

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**Abstract** Multi-agent systems have been studied in various contexts of both application and theory. We take Dynamic Epistemic Logic (DEL), one of the formalisms designed to reason about such systems, as the foundation of the language we will build.

BioAmbient calculus is an extension of  $\pi$ -calculus, developed largely for applications to biomolecular systems. It deals with ambients and their ability to communicate and to execute concurrent processes while moving.

In this paper we combine the formalism of Dynamic Epistemic Logic together with the formalism of BioAmbient Calculus in order to reason about knowledge maintained and gained upon process transitions. The motivation lies in developing a language that captures locally available information through assignment of knowledge, with potential application to biological systems as well as social, virtual, and others.

We replace the ambients of BioAmbient Calculus with agents, to which we attribute knowledge, and explore the parallels of this treatment. The resulting logic describes the information flow governing mobile structured agents, organized hierarchically, whose architecture (and local information) may change due to actions such as *communication*, *merging* (of two agents), *entering* (of an agent into the inner structure of another agent) and *exiting* (of an agent from the structure of another). We show how the main axioms of DEL must be altered to accommodate the informational effects of the agents' dynamic architecture.

## 1 Introduction

We develop a formalism *PADEL* suited for talking about various multi-agent systems. In particular, we discuss previous and potential applications to systems of

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molecular biology, though the language is not limited to this. We develop the notion of an *agent*, which can refer to an entire system or a subsystem thereof, seen as *informational* (and *information-acquiring*) systems. Information locally available to a given system is treated as *knowledge* and the flow and exchange of information between systems as *dynamics of knowledge* in a multi-agent setting. For all of the above, we rely on a formalism derived from Dynamic Epistemic Logic and BioAmbient Calculus.

We assume the following things about the architecture of these agents: First, the number of agents (and thus subagents) is always finite. Second, they are nested in a dynamic tree structure (with no loops).

In addition to typical communication actions, such as sending and receiving information or public announcements, we consider three specific actions which involve mobility: entering, exiting, and merging. The formalisation and the specific rules for the latter are inspired largely by Luca Cardelli's developments in BioAmbient Calculus<sup>1</sup>, which aims to formalize information flow in systems of molecular biology.

## 2 The formalism and motivation of $\mathcal{PADEL}$

At a given state, an agent is defined by an assignment of concurrent processes, and in a given process there can occur agents, capabilities, or other non-agent, non-capability processes.

### 2.1 Basic Definitions

Let  $\mathcal{A}$  be a finite set of agents and  $\mathcal{A}c$  a finite set of atomic actions.

$$\text{An agent } A \in \mathcal{A} \text{ occurs in a process } P, \text{ or } A \sqsubseteq P, \text{ iff } \begin{cases} A \sqsubseteq A \\ A \sqsubseteq P \Rightarrow A \sqsubseteq P \mid Q \\ A \sqsubseteq P \Rightarrow A \sqsubseteq Q \mid P \\ A \sqsubseteq P \Rightarrow A \sqsubseteq a.P \end{cases}$$

where  $P \mid Q$  denotes two processes running in parallel and  $a.P$  denotes a capability  $a$ , which, if executed, will initiate a process  $P$ .

We define  $\sqsubseteq^+$  as the transitive closure of  $\sqsubseteq$ :

$A \sqsubseteq^+ P \Leftrightarrow \exists$  a chain  $P_0, P_1, \dots, P_n$  of processes s.t.  $n > 0$ ,  $A = P_0$ ,  $P = P_n$ , and  $P_{i-1} \sqsubseteq P_i$ , for all  $i \leq n$ .

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<sup>1</sup> See [10] and [11]

An agent  $A \in \mathcal{A}$  is a *subagent* of  $P$ , or  $A < P$ , iff 
$$\begin{cases} A < A \mid P \\ A < P \mid A \\ A < P \Rightarrow A < P \mid Q \\ A < P \Rightarrow A < Q \mid P \end{cases}$$

**Definition** A state  $s$  is an assignment of Processes to Agents,  $s : \text{Agents} \rightarrow \text{Processes}$ , such that for every two distinct agents  $A, B \in \mathcal{A}$  and for any agent  $C \in \mathcal{A}$ :

$$C \sqsubseteq s(A), C \sqsubseteq s(B) \Rightarrow A = B \text{ and } A \not\sqsubseteq^+ s(A) \quad (1)$$

That is, agent  $C$  cannot simultaneously occur in a process assigned to two different agents and an agent cannot occur in a process assigned to itself.

For a given state  $s$  and two agents  $A, B$ , we define  $A <_s B \stackrel{\text{def}}{=} A < s(B)$ . We read  $A <_s B$  as saying that  $A$  is a subagent of (agent)  $B$  in state  $s$ .

### Consequences

$$A <_s B \Rightarrow A \sqsubseteq s(B). \quad (2)$$

From (1) and (2), it follows also that:

$$C <_s A, C <_s B \Rightarrow A = B \quad (3)$$

In other words, assignments  $s(A)$  and  $s(B)$  for different agents  $A \neq B$  must contain no agents in common.

**Definition** We define  $<_s^+$  as the transitive closure of  $<_s$ , and call it the *iterative subagent relation* at state  $s$ , while referring to  $<_s$  as the *one-step subagent relation*.

$A <_s^+ B \Leftrightarrow \exists$  a finite chain  $A_0, A_1, \dots, A_n$   
s.t.  $n > 0, A = A_0, B = A_n$ , and  $A_{i-1} <_s A_i$ , for all  $i \leq n$ .

Consequence 3 in turn disallows loops in the tree of agents:

**Proposition [Tree Property]** For  $A \neq B$ :

$$C <_s^+ A, C <_s^+ B \Rightarrow A <_s^+ B \vee B <_s^+ A. \quad (4)$$

*Proof.* We prove this by induction on the length of the chain. By the hypothesis, there must exist two chains, where  $A \neq B$ :

$C = X_1 <_s X_2 <_s \dots X_{i-1} <_s X_i <_s X_{i+1} <_s \dots <_s X_n = A$  and

$C = Y_1 <_s Y_2 <_s \dots Y_{i-1} <_s Y_i <_s Y_{i+1} <_s \dots <_s Y_m = B$ .

WLOG, suppose  $n \leq m$  (the case for  $m < n$  is similar).

Then, by (3),  $X_2 = Y_2$ , and again by (3),  $X_3 = Y_3$ , and so on until  $X_n = Y_n$ ,  $\Rightarrow A = Y_n$ .

Now, if  $n = m$ , then  $A = Y_m = B$ , contradicting the fact that  $A$  and  $B$  were assumed to be distinct.

If  $n < m$ , then  $A = Y_n <_s Y_{n+1} <_s \dots <_s Y_m = B$  and we have shown that  $A <_s^+ B$ .  $\square$

Given a finite set  $\mathcal{A}$  of agents, denoted by  $A, B, C, A_1, \dots, A_n$ , and given a finite set  $\mathcal{A}c$  of atomic actions, denoted by  $a, a_i, \bar{a}$ , we combine the syntax of BioAmbient Process Algebra and DEL, adding only the atomic sentence  $A <^+ B$ , and define the

sentences of propositional logic together with the *one – step* subagent relation.  $\varphi, \psi$  are formulae and  $p$  are propositional sentences in the language:

**Table 1** Syntax and Definitions

Assume  $A, B, C$  are distinct agents. Then:

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$$\begin{aligned}
 P &::= \mathbf{0} \mid A \mid (P \mid P) \mid \Sigma_i a_i.P_i \\
 \varphi &::= A <^+ B \mid \neg\varphi \mid \varphi \wedge \psi \mid K_A\varphi \mid DK_{A_1, \dots, A_n}\varphi \mid [\alpha]\varphi \\
 \alpha &\in \{(a_A, \bar{a}_B)\} \\
 a, a_i, \bar{a} &\in \{\varphi?, \varphi!, \text{enter}, \text{accept}, \text{exit}, \text{expel}, \text{merge}+, \text{merge}-\}
 \end{aligned}$$

$$\begin{aligned}
 \varphi \vee \psi &:\stackrel{def}{=} \neg(\neg\varphi \wedge \neg\psi) \\
 \varphi \Rightarrow \psi &:\stackrel{def}{=} \neg(\varphi \vee \neg\psi) \\
 \varphi \Leftrightarrow \psi &:\stackrel{def}{=} (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi) \\
 A < C &:\stackrel{def}{=} A <^+ C \wedge \bigwedge_{B \in \mathcal{A}} \neg(A <^+ B \wedge B <^+ C) \\
 < \alpha > \varphi &:\stackrel{def}{=} \neg[\alpha]\neg\varphi \\
 \top &:\stackrel{def}{=} p \vee \neg p, \text{ for some fixed } p \\
 \perp &:\stackrel{def}{=} p \wedge \neg p, \text{ for some fixed } p
 \end{aligned}$$


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## 2.2 Actions

The set  $\mathcal{A}c$  of atomic actions is finite. Similar to the notions of *executability*, or *precondition*, in DEL, agent  $A$  must have the capability  $a.P$  included in the processes assigned to it at the initial state in order for  $a_A$  to take place (agent  $A$  executing action  $a$ ).

The capabilities each agent is assigned at a given state are expressed as a non-deterministic sum of atomic actions  $\sum_i a_i.P_i$ , followed by the processes that would initiate as a result of  $A$  performing a given atomic action.

### 2.2.1 State Transitions

Following the Bioambient improvement on Ambient Calculus, we only allow suitable action *pairs* to induce state transitions. A cell has to accept a virus that is trying to enter, just like an announcement must be heard in order for it to affect an agent's knowledge.

We define actions  $\alpha$  as dual pairs of atomic actions, which form a finite set  $i\mathcal{A}c$ :

$$\alpha = (a, \bar{a}) \in \mathcal{A}c \times \mathcal{A}c = i\mathcal{A}c.$$

We use  $B :_s \alpha$  to denote agent  $B$ 's participation in action  $\alpha$  at state  $s$ . For  $\alpha = (a_A, \bar{a}_C)$ :

$$B :_s \alpha \stackrel{def}{=} A \leq_s^+ B \vee C \leq_s^+ B \text{ and } \exists s' \text{ s.t. } s \xrightarrow{\alpha} s'$$

We define four types of actions, of which three involve a *one – step* superagent  $E$  whose state assignment is crucial to the executability of the action (see Figure 1).

For any agents  $A, C, E$  that are distinct:

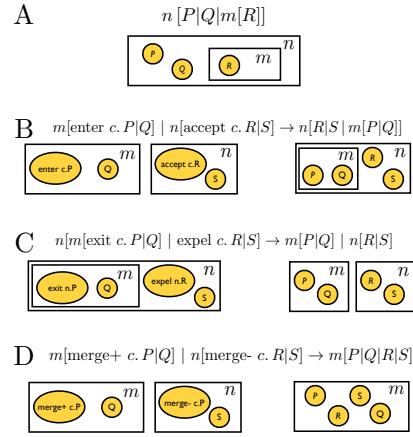
$$\alpha_I = (\varphi?_A, \varphi!_C),$$

$$\alpha_{II} = (enter_A, accept_C, E),$$

$$\alpha_{III} = (exit_A, expel_C, E),$$

$$\alpha_{IV} = (merge+_A, merge-_C, E)$$

**Fig. 1** Motivated by work of Luca Cardelli (see [9]-[11]), this figure depicts the application of this language to molecular biology. B, C, and D show the change in structure of processes and subprocesses as a result of acting on dual capabilities (Types II, III, IV, respectively), separated by no more than two "membranes." In  $\mathcal{P}\mathcal{A}\mathcal{D}\mathcal{E}\mathcal{L}$ , we can think of each "membrane" with all its contents as a unique agent.



We now define the state transitions for the four different types of actions.

**Type I** For  $\alpha_I = (a_A, \bar{a}_C)$ , where  $(a, \bar{a}) = (\varphi?, \varphi!)$ :

$$s \xrightarrow{\alpha_I} s' \text{ iff}$$

$$\exists a, a_i, P, P_i, Q \text{ such that } s(A) = \sum_i a_i.P_i + a.P \mid Q,$$

$$\exists \bar{a}, c_j, R, R_j, S \text{ such that } s(C) = \sum_j c_j.R_j + \bar{a}.R \mid S,$$

$$s'(A) = P \mid Q, s'(C) = R \mid S, s'(X) = s(X), \text{ for all } X \neq A, C.$$

This is the only type of action that does not change the structure of the tree.

**Type II** For  $\alpha_{II} = (a_A, \bar{a}_C, E)$  where  $a = enter$ ,  $\bar{a} = accept$ :

$$s \xrightarrow{\alpha_{II}} s' \text{ iff}$$

$$\exists a, a_i, P, P_i, Q \text{ such that } s(A) = \sum_i a_i.P_i + a.P \mid Q,$$

$$\exists \bar{a}, c_j, R, R_j, S \text{ such that } s(C) = \sum_j c_j.R_j + \bar{a}.R \mid S,$$

$\exists \Gamma$ , a process, such that  $s(E) = A \mid C \mid \Gamma$ , and  
 $s'(A) = P \mid Q$ ,  $s'(C) = A \mid R \mid S$ ,  $s'(E) = C \mid \Gamma$ ,  $s'(X) = s(X)$ , for all  $X \neq A, C, E$ .

After  $\alpha_{II}$  state transition, agent  $C$  is assigned a new agent, while agent  $E$  – the initial superagent of both  $C$  and  $A$  – is stripped of the *one – step* subagent  $A$ :

$$\begin{array}{ccc} E & \xrightarrow{\alpha_{II}} & E \\ \widehat{A \ C} & & \mid \\ & & C \\ & & \mid \\ & & A \end{array}$$

**Type III** For  $\alpha_{III} = (a_A, \bar{a}_C, E)$  where  $a = \text{exit}$ ,  $\bar{a} = \text{expel}$ :

$s \xrightarrow{\alpha_{III}} s'$  iff

$\exists a, a_i, P, P_i, Q$  such that  $s(A) = \sum_i a_i.P_i + a.P \mid Q$ ,

$\exists \bar{a}, c_j, R, R_j, S$  such that  $s(C) = \sum_j c_j.R_j + \bar{a}.R \mid A \mid S$ ,

$\exists \Gamma$ , a process, such that  $s(E) = C \mid \Gamma$ , and

$s'(A) = P \mid Q$ ,  $s'(C) = R \mid S$ ,  $s'(E) = C \mid A \mid \Gamma$ ,  $s'(X) = s(X)$ , for all  $X \neq A, C, E$ .

After  $\alpha_{III}$  this state transition, the effect is exactly opposite to that of transitions by actions of Type II:

$$\begin{array}{ccc} E & \xrightarrow{\alpha_{III}} & E \\ \mid & & \widehat{A \ C} \\ C & & \\ \mid & & \\ A & & \end{array}$$

**Type IV**  $\alpha_{IV}$ , defined as  $(a_A, \bar{a}_C, E)$  where  $a = \text{merge+}$ ,  $\bar{a} = \text{merge-}$ :

$s \xrightarrow{\alpha_{IV}} s'$  iff

$\exists a, a_i, P, P_i, Q$  such that  $s(A) = \sum_i a_i.P_i + a.P \mid Q$ ,

$\exists \bar{a}, c_j, R, R_j, S$  such that  $s(C) = \sum_j c_j.R_j + \bar{a}.R \mid S$ ,

$\exists \Gamma$ , a process, such that  $s(E) = A \mid C \mid \Gamma$ , and

$s'(A) = P \mid Q \mid R \mid S$ ,  $s'(C) = \mathbf{0}$ ,  $s'(E) = A \mid \Gamma$ ,  $s'(X) = s(X)$ , for all  $X \neq A, C, E$ .

$$\begin{array}{ccc} E & \xrightarrow{\alpha_{IV}} & E \\ \widehat{A \ C} & & \mid \\ & & A \end{array}$$

The following validities follow immediately from the definitions, where  $\langle \alpha_i \rangle \top$  denotes executability of  $\alpha_i$ , and  $[\alpha_i]\varphi$  denotes a statement  $\varphi$  that holds true after action  $\alpha_i$ :

**Table 2** Consequences of Action Definitions

Assume  $A, C, E$  are distinct agents. Then:

$\langle \alpha_{II} \rangle \top \Rightarrow A < E \wedge C < E$
$\langle \alpha_{III} \rangle \top \Rightarrow A < C \wedge C < E$
$\langle \alpha_{IV} \rangle \top \Rightarrow A < E \wedge C < E$
$[\alpha_{II}]A < C$
$[\alpha_{III}]A < E$
$[\alpha_{IV}]\neg C < E$

### 2.3 Indistinguishability Relations on States and Actions

As in Epistemic Logic (EL), we define indistinguishability of two states for a particular agent  $A$ , denoted by  $\sim_A$ , in order to reason about knowledge.

**Definition**  $s \sim_A s'$  iff  $s(X) = s'(X)$ , for all  $X \leq_s^+ A$ .

That is, two states are equivalent for agent  $A$  if and only if they are indistinguishable for  $A$  and all of its subagents, as assigned at state  $s$ . Two states can be indistinguishable for a group of agents  $B_1, \dots, B_n$  if none of them can distinguish between these states:

**Definition**  $s \sim_{B_1, \dots, B_n} s'$  iff  $s(X) = s'(X)$ , for all  $X \leq_s^+ B_i$ , for all  $1 \leq i \leq n$ .

We define indistinguishability of actions:

**Definition** [Equivalence of actions] Here  $s$  represents any state in the history.

$$\alpha \sim_s^A \alpha' \Leftrightarrow \begin{cases} \text{either } A :_s \alpha \text{ and } \alpha = \alpha' \\ \text{or } \neg A :_s \alpha \text{ and } \neg A :_s \alpha' \end{cases}$$

If  $A$  is a participant in  $\alpha$ , then it would certainly be able to differentiate between taking part in two different actions  $\alpha$  and  $\alpha'$ , unless they were actually the same. On the other hand, if  $A$  does not participate in either  $\alpha$  or  $\alpha'$ , then both actions appear equivalent to  $A$ . This implies that  $A$  is a subagent of both agents executing  $\alpha$ .

## 3 Semantics

We will evaluate logical formulas on *histories*, which are sequences of states and actions (representing possible histories of a system). However, in order to define the semantics for epistemic and dynamic modalities, we need to define appropriate (epistemic) indistinguishability relations and (dynamic) transition relations on histories, by lifting to histories the corresponding state relations.

### 3.1 Relations on Histories

To ensure that our knowledge is accumulative, as in DEL, we must expand the language to include Perfect Recall and extend equivalence relations to state transitions and to previous states. For this we define histories and develop axioms based on histories rather than states.

We define a *history*  $h$  as a sequence of alternating states and actions:

$$h = (s_0, \alpha_0, s_1, \alpha_1, \dots, s_{n-1}, \alpha_{n-1}, s_n) \text{ s.t.} \\ s_i \xrightarrow{\alpha_i} s_{i+1} \text{ for all } i < n.$$

- $(h, \alpha, t) := (s_0, \alpha_0, s_1, \alpha_1, \dots, s_{n-1}, \alpha_{n-1}, s_n, \alpha, t)$  iff  $s_n \xrightarrow{\alpha} t$
- $|h|$  denotes the size of the history, equal to the number of state-action pairs in the history, not counting the final state
- $last(h) = s_n$ . We use the convention of  $h \models \varphi$  iff  $last(h) \models \varphi$

We extend the notion of state indistinguishability to history indistinguishability for an agent  $A$ .

**Definition** [Equivalence of histories]

Let  $h = (s_0, \alpha_0, s_1, \alpha_1, \dots, s_i, \alpha_i, \dots, \alpha_{n-1}, s_n)$  and let  $h' = (s'_0, \alpha'_0, s'_1, \alpha'_1, \dots, s'_i, \alpha'_i, \dots, \alpha'_{n-1}, s'_n)$ , then

$$h \overset{A}{\sim} h' \Leftrightarrow \forall i \in \{0, 1, 2, \dots, n\} : |h| = |h'| \text{ and } s_i \overset{A}{\sim} s'_i \text{ and } \alpha_i \overset{A}{\sim}_{s_i} \alpha'_i$$

**Definition** [History transition]

For two histories  $h, h'$ ,

$$h \xrightarrow{\alpha} h' \text{ iff } \exists t \text{ s.t. } h' = (h, \alpha, t).$$

**Proposition** [Perfect Recall] This follows from the definitions above and ensures uniqueness of history transitions.

$$h' \sim_C h'', h' = (h_1, \alpha, s'), h'' = (h_2, \beta, s'') \Rightarrow |h_1| = |h_2|, h_1 \sim_C h_2, \alpha \sim_C \beta.$$

**Proposition** Indistinguishable histories for an agent remain indistinguishable for its subagents in the last state:

$$h \sim_C h', A <_{last(h)}^+ C \Rightarrow h \sim_A h'$$

*Proof.*  $s_i(X) = s'_i(X)$ , for all  $i$ , for all  $X \leq^+ C$  implies the same for  $X \leq^+ A$  since  $A$  is a subagent of  $C$ .

Now, for each  $\alpha_i \sim_C \alpha'_i$  in the histories, if  $C$  is not a participant of  $\alpha$  and they appear to be the same, then by definition of participation the same holds for  $A$  since it is a subagent.

If  $C :_{s_i} \alpha$ , then  $\alpha_i = \alpha'_i$ . In this case, regardless of whether or not  $A$  participates in  $\alpha$ , the two appear the same to it.  $\square$

The definition for equivalence of histories for a group of agents is similar:

$$h \sim_{B_1, \dots, B_n} h' \stackrel{def}{=} h \sim_{B_1} h' \cap \dots \cap h \sim_{B_n} h' \quad (5)$$



### 3.2 Semantics

The semantics of our language is embodied by a satisfaction relation  $\models$  between histories and logical formulas, which is defined by the inductive clauses in Table 3. The definition is by induction on formulas. For  $A, B, B_1, \dots, B_n$ , distinct,  $\in \mathcal{A}$ :

**Table 3** Semantics

$h \models A <^+ B$	iff $A <_{last(h)}^+ B$
$h \models \neg\varphi$	iff $h \not\models \varphi$
$h \models \varphi \wedge \psi$	iff $h \models \varphi$ and $h \models \psi$
$h \models K_A \varphi$	iff $\forall (h' \sim_A h) : h' \models \varphi$
$h \models DK_{B_1, \dots, B_n} \varphi$	iff $\forall (h' \sim_{B_1, \dots, B_n} h) : h' \models \varphi$
$h \models [\alpha] \varphi$	iff $\forall h \xrightarrow{\alpha} h' : h' \models \varphi$

## 4 Proof System

We use axioms and rules of inference from propositional logic and those of DEL<sup>2</sup>, together with those specific to our formalism. In addition, we outline reduction laws, with select proofs. In this section,  $A, B, C, X, Y, B_1, \dots, B_n, A_1, \dots, A_n$  are agents  $\in \mathcal{A}$ .

**Table 4** Axioms of Knowledge

$\vdash DK_A \varphi$	$\Leftrightarrow K_A \varphi$	G1
$\vdash K_A \varphi$	$\Rightarrow DK_{A, B_1, \dots, B_n} \varphi$	KtoDK
$\vdash A <^+ C$	$\Rightarrow K_C(A <^+ C)$	KOwn
$\vdash A <^+ C \wedge C <^+ B$	$\Rightarrow DK_{B, A_1, \dots, A_n}(A <^+ C)$	DKOwn
$B_1, \dots, B_n <^+ A \wedge DK_{B_1, \dots, B_n} \varphi$	$\Rightarrow K_A \varphi$	KfromDK

*Proof.* [KOwn] The right hand side of the statement is equivalent to  $\forall h'(h \sim_C h' \Rightarrow h' \models A <^+ C)$ . By the definition of equivalence, we have that  $\forall i, s_i(X) = s'_i(X)$ , for all  $X \leq^+ C$ , which implies that state assignments, for all states in histories  $h, h'$  will be the same for  $C$  and its subagents.

<sup>2</sup> These include the Necessitation and the Modus Ponens rules of inference, as well as KT45 axioms and all tautologies of propositional logic. See [13] for more description.

But then  $last(h)(X) = last(h')(X)$  will also hold true for  $X = C$  and  $X = A$  and all agents in between them, thus satisfying  $h' \models A <^+ C$ .  $\square$

Axioms R, Trans, and Tree reveal the loop-less tree structure of agents.

**Axiom R**  $\vdash \neg A <^+ A$

**Axiom Trans**  $\vdash A <^+ B \wedge B <^+ C \Rightarrow A <^+ C$

**Axiom Tree**  $\vdash (X <^+ A \wedge X <^+ B) \Rightarrow (A <^+ B \vee B <^+ A)$

*Proof.* [Axiom Tree] For  $s = last(h)$ , the statement is semantically equivalent to  $X <_s^+ A$  and  $X <_s^+ B$ , for some state  $s$ . But then by (4), we guarantee that  $B <_s^+ A$  or  $A <_s^+ B$ , which is semantically equivalent to the desired result.  $\square$

We now explore reduction laws involving the dynamic modality.

**Partial Functionality Axiom**  $[\alpha]\neg\varphi \Leftrightarrow (\langle \alpha \rangle \top \Rightarrow \neg[\alpha]\varphi)$

That is, the transition induced by  $\alpha$ , if it exists, goes to a unique next state: if  $h \xrightarrow{\alpha} h'$  and  $h \xrightarrow{\alpha} h''$ , then  $h' = h''$ . This ensures uniqueness of transition.

The Preservation of Facts axiom of DEL demands several versions for the different types of actions (see Table 5).

**Table 5** Preservation of Facts Axioms<sup>a</sup>

$[\alpha_I]\varphi$	$\Leftrightarrow (\langle \alpha_I \rangle \top \Rightarrow \varphi)$	PF1
For $X \neq A$ :		
$[\alpha_{II}]X < Y$	$\Leftrightarrow (\langle \alpha_{II} \rangle \top \Rightarrow X < Y)$	PF2a
For $Y \neq E, C$ :		
$[\alpha_{II}]A < Y$	$\Leftrightarrow (\langle \alpha_{II} \rangle \top \Rightarrow A < Y)$	PF2b
For $X \neq A$ :		
$[\alpha_{III}]X < Y$	$\Leftrightarrow (\langle \alpha_{III} \rangle \top \Rightarrow X < Y)$	PF3a
For $Y \neq E, C$ :		
$[\alpha_{III}]A < Y$	$\Leftrightarrow (\langle \alpha_{III} \rangle \top \Rightarrow A < Y)$	PF3b
$(X < C)$	$\Rightarrow [\alpha_{IV}](X < A)$	PF4a
For $X \neq C$ :		
$\neg(X < A) \Rightarrow ([\alpha_{IV}]X < Y \Leftrightarrow (\langle \alpha_{IV} \rangle \top \Rightarrow X < Y))$		PF4b

<sup>a</sup> Note that the Consequences outlined in Table 2 also belong to this category of reduction laws.

*Proof.* [PF4a] We unwrap the definition for Type IV action, found in 2.2.1, where  $s = \text{last}(h)$ . It follows:

If  $X < C$  at  $\text{last}(h)$ , then  $X \sqsubseteq S$  (occurs in process  $S$ ).

Since  $s'(X) = s(X)$ , for all  $X \neq A, C, E$ , then  $X$  still occurs in  $S$  at  $s'$ .

Since  $s'(A)$  is assigned process  $S$ , where  $X$  occurs, then  $X$  must be a subagents of  $A$  at  $s'$ .  $\square$

Similarly, the Action-Knowledge reduction laws are expanded for specificity (see Table 6).

**Table 6** Action-Knowledge Axioms

For $X = A, C$ :	$[\alpha_I]K_X \varphi \Leftrightarrow (\langle \alpha_I \rangle \top \Rightarrow K_X[\alpha_I]\varphi)$	AcKn1a
$(A <^+ X \vee C <^+ X) \Rightarrow$	$[\alpha_I]K_X \varphi \Leftrightarrow (\langle \alpha_I \rangle \top \Rightarrow K_X[\alpha_I]\varphi)$	AcKn1b
$C <^+ X \Rightarrow$	$[\alpha_{II}]K_C \varphi \Leftrightarrow (\langle \alpha_{II} \rangle \top \Rightarrow DK_{A,C}[\alpha_{II}]\varphi)$	AcKn2a
	$[\alpha_{II}]K_X \varphi \Leftrightarrow (\langle \alpha_{II} \rangle \top \Rightarrow K_X[\alpha_{II}]\varphi)$	AcKn2b
	$[\alpha_{II}]K_A \varphi \Leftrightarrow (\langle \alpha_{II} \rangle \top \Rightarrow K_A[\alpha_{II}]\varphi)$	AcKn2c
For $X = A, C$ :	$[\alpha_{III}]K_X \varphi \Leftrightarrow (\langle \alpha_{III} \rangle \top \Rightarrow K_X[\alpha_{III}]\varphi)$	AcKn3a
$A <^+ X \Rightarrow$	$[\alpha_{III}]K_X \varphi \Leftrightarrow (\langle \alpha_{III} \rangle \top \Rightarrow K_X[\alpha_{III}]\varphi)$	AcKn3b
$A <^+ X \Rightarrow$	$[\alpha_{IV}]K_A \varphi \Leftrightarrow (\langle \alpha_{IV} \rangle \top \Rightarrow DK_{A,C}[\alpha_{IV}]\varphi)$	AcKn4a
	$[\alpha_{IV}]K_X \varphi \Leftrightarrow (\langle \alpha_{IV} \rangle \top \Rightarrow K_X[\alpha_{IV}]\varphi)$	AcKn4b
$(X <^+ A \wedge X <^+ C) \Rightarrow$	$[\alpha]K_X \varphi \Leftrightarrow \bigwedge_{\beta \in i\alpha'c^+, \beta \sim_X \alpha} (\langle \alpha \rangle \top \Rightarrow [\beta]\varphi)$	AcKnNP

Note that the final rule in Table 6 is for non-participants of any action  $\alpha$ . All proofs are achieved by a counterfactual argument of “chasing the diagram,” though we omit them here.

**Theorem** The proof system for  $\mathcal{PADEL}$  is sound.

*Proof.* In order to show soundness, all axioms in the system must be valid. For all axioms presented in gray boxes, validity was either proved in the text or it follows from the semantic definitions.  $\square$

**Theorem** [Model-checking] The model-checking problem for  $\mathcal{PADEL}$  is decidable on finite models.

*Proof.* Given a model  $M$  with a countable set of histories  $h$  and formula  $\varphi$ , the axioms and rules of inference are sufficient to decide whether or not  $\varphi$  is satisfiable at  $M, h$ , since we have provided axioms for all syntactic combinations of terms  $\varphi$  can have.  $\square$

**Corollaries** The following are semantically valid consequences of axioms and rules of inference:

- $\vdash A < B \Rightarrow A <^+ B$
- $\vdash X < A \Rightarrow \neg X < B$
- $\vdash A <^+ C \wedge C <^+ B_1, \dots, B_n \Rightarrow DK_{B_1, \dots, B_n}(A <^+ C)$
- $\vdash A <^+ C \wedge C <^+ B \Rightarrow K_B(A <^+ C)$
- $\vdash B_1, \dots, B_n <^+ A \wedge DK_{B_1, \dots, B_n} A \varphi \Rightarrow K_A \varphi$
- $(X <^+ A) \Rightarrow [\alpha_{III}] \neg(X <^+ C)$

## 5 Conclusion

We have thus developed a sound, decidable language  $\mathcal{P}\mathcal{A}\mathcal{D}\mathcal{E}\mathcal{L}$  based on a nested tree structure of a finite number of agents, which are defined by concurrent processes, subagents and capabilities, Furthermore, we developed the notion of knowledge and distributed knowledge for agents based on

1. the current *state* of an agent, which captures its current *one – step* subagents and its current capabilities for future interactions
2. the current *state* of all of its *iterative* subagents. This encodes a principle of monotonicity of information: all information carried by a subagent is available to any of its superagents
3. the memory of an agent, encoded in a history that each agent perceives differently. Following the premises of DEL, information is never lost and contradictory knowledge is never acquired.

The presented axiomatization allows one to reason about knowledge and change in knowledge of agents executing actions, as well as their subagents and superagents. Further applications to biological systems remain to be explored, in particular seeking to define “knowledge,” as described by indistinguishabilities, for a given biological unit. It also remains to investigate whether the system is complete.

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