

# The Muddy Children Puzzle : An alternative scenario

Anya Yermakova

Instead of presenting the classical coin-toss or muddy children scenarios as an example, we present one almost identical to the muddy children, where the agents are countries. We take agent A to be Russia, agent B to be the United States of America, and agent C to be the United Kingdom. Note that this scenario is entirely hypothetical and does not draw to any events in the real world.

Upon receiving a complaint from Canada, the UN General informed the three countries listed above that signs of radioactivity have been measured on Canadian land, and the location of these measurements indicates that the source of this radioactivity must be either on Russian-run, US-run, or UK-run research territories in Canada. Since these measurements are quite severe, it is a true fact that at least one of these territories must be in possession of harmful radioactive material.

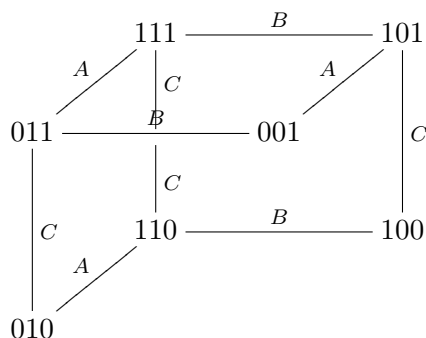
All three countries are (honestly) unaware of any such production, and immediately deny any responsibility. The UN General, however, relying on that honesty, instructs each nation to send a team of workers to thoroughly inspect two of the three territories, leaving out the one run by itself. That is, Russia is instructed to inspect UK and US territories, US is instructed to inspect Russian and UK territories, and the UK is instructed to inspect Russian and US territories. They agree, with the promise to keep in mind that **at least one** of the territories must contain radioactive material.

Let 0 represent that a territory is “clean” - lacks radioactive material, and let 1 represent that a territory is “dirty” – has radiation material. We then have seven possible states:  $S = \{(001), (010), (100), (110), (011), (101), (111)\}$

At the onset, we have the following indistinguishability relations, since each country will not be able to inspect its own territory:

$(011) \sim_A (111), (001) \sim_A (101), (010) \sim_A (110),$   
 $(101) \sim_B (111), (100) \sim_B (110), (001) \sim_B (011),$   
 $(111) \sim_C (110), (101) \sim_C (100), (010) \sim_C (011).$

Thus we have a Kripke model, depicted below.



Now, the countries comply with the instructions from the UN and discover the following: Russia finds radioactive debris on US territory, US finds radioactive debris on Russian territory, and the UK finds radioactive debris on both Russian and US territories. The omniscient reader understands that the actual state of the world is (110).

Instead of simply reporting on what they found, the UN General assembles a meeting and asks only one question for each representative from the three countries: “Do you know if your own territory has radioactive material on it?” All three announce “no.” Let us see how the Kripke model changes due to that announcement.

Russia, coming to the meeting, and expecting this question, reasons as follows: Both us and the US know that the UK is clean of waste, because we have both searched its territories. If the USA admits to having waste (responds ‘yes’ at the meeting), that must be because they found no waste on our (Russian) territories and deduced that it must be

on theirs since at least one of us three must have waste. If, however, the USA says “no,” then Russian territory must have waste.

The USA reasons in the same manner: If Russia admits to having waste (responds ‘yes’ at the meeting), that must be because they found no waste on our (USA) territories and deduced that it must be on theirs since at least one of us three must have waste. If, however, Russia says “no,” then US territory must have waste.

The UK knows that both will respond with “no” at the first meeting, but knows that it will figure out whether or not the UK territory is waste-free only if the General calls a second meeting.

Since all three countries respond with “no,” Russia eliminates the states where it is clean (thus losing all its indistinguishability relations), the US eliminates the states where it is clean (thus also losing all its indistinguishability relations). We thus update the Kripke structure to:

$$\begin{array}{c} 111 \\ | \\ C \\ | \\ 110 \end{array}$$

Both Russia and the USA have lost their indistinguishability relations in the Kripke model, showing that they now know which state of the world they are in. The UK, however, is still uncertain.

Anxious to know whether or not all three countries are actually bringing radioactive damage on Canadian territory, the UN General calls a second meeting, asking the same question: “Do you know if your own territory has radioactive material on it?” Both Russia and the USA respond “yes.” The UK representative jumps with joy, as those response free his country from fault. Here is why:

If the UK territory was clean, then the US would have found waste only on Russian territory. Upon hearing “no” in the first meeting from the Russians, the US would only be able to say “yes” in the second if the UK territory was waste-free. This is because the General clearly stated that at least one of them must have radioactive material. Hearing “no” from the Russians signified to the USA that they themselves are dirty, since otherwise Russia would be the only country guilty of contaminating Canada, and this reasoning is only possible if the UK territory is waste-free. Otherwise, even in the second meeting the USA (and Russia) would have said “no,” as the lack of knowledge of the other countries did not help them deduce their own state.

Thus, the UN General found no need to call a third meeting, as the excited reaction of the UK representative after the positive announcements of the Russians and the Americans was indicative enough of the current state of the world.

In this way, the UN General forced the US and Russia to admit its fault by inductive reasoning, as opposed to actually going and searching for the evidence. Perhaps he was also afraid of each country finding the evidence and then hiding it, or taking measures to hide its discovery. Since every country, however, had full interest to do as thorough search as possible on foreign territory, he was certain that their findings would be truthful.